APPENDIX 1

The distribution of the Nyquist filtering on two filters therefore commonly called Nyquist root filters or, in short, half-Nyquist filters, must take account of the constraints related to the ISI, the frequency specifications, the linearity of the phase and what is called the matched pair property. In this appendix, we define these different notions assuming, to simplify the notation, that $T = 1$. Furthermore, in the context of the embodiment, we shall deal only with the case of the finite pulse response filters (FIR).

1 - Intersymbol interference (ISI)

Let $x(n)$ be the input signal and $X(z)$ its z transform, namely $X(z) = \sum_n x(n)z^{-n}$ and let it be agreed that all the signals of the transmission system will be referenced according to the same principle. The null ISI constraint amounts to the dictating, when there is no noise, of the output signal referenced $S(z)$ which is identical, except for a delay, to $X(z)$.

Let $Y(z)$ be the output signal from the sender. It can be written as follows:

$$Y(z) = F_T(z)X(z^M). \tag{1}$$

At output of the reception filter, the signal referenced $U(z)$ has the following expression:

$$U(z) = F_T(z)F_R(z)X(z^M). \tag{2}$$

The output signal is therefore quite simply:

$$S(z) = \frac{1}{M}\sum_{k=0}^{M-1} U(z^{\frac{1}{M}}w^k) = \left[\frac{1}{M}\sum_{k=0}^{M-1} F_T(z^{\frac{1}{M}}w^k)F_R(z^{\frac{1}{M}}w^k)\right] X(z), \tag{3}$$

where $w = e^{\frac{2j\pi}{M}}$ is the Mth root of unity. The null ISI condition can then be summarized in the following equation:

$$\frac{1}{M}\sum_{k=0}^{M-1} F_T(z^{\frac{1}{M}}w^k)F_R(z^{\frac{1}{M}}w^k) = z^{-d}, \tag{4}$$

where d corresponds to the delay introduced by the two filters.

Let $P(z)$ be the filter produced, i.e. $P(z) = F_T(z)F_R(z)$. In a formal method commonly used in with multi-rate signal processing, the equation (4) can then be written as follows:

$$P(z)\downarrow_M = z^{-d}. \tag{5}$$

A filter *P(z)* that verifies this relationship is a Nyquist filter or again an Mth band filter.

Let n_T and n_R be the respective orders of the sending and reception FIR filters. The filter P is therefore an $n_P = n_T + n_R$ order filter and can be put in the following form:

$$P(z) = \sum_{n=0}^{n_P} p(n) z^{-n}. \tag{6}$$

If the equation (5) is not verified, it is commonly the "distance" that is verified with respect to the property by either of the following expressions:

$$D_1 = \frac{1}{|\, p(d) \,|} \sum_{kM \neq d} |\, p(d - kM) \,|, \tag{7}$$

where

$$D_2 = \frac{1}{p^2(d)} \sum_{kM \neq d} p^2(d - kM). \tag{8}$$

2. The frequency specifications

Figure 2 shows a typical frequency specification for digital transmission filters:

- The roll-off factor defines, for these lowpass filters, the passband by

$$[0,\ \omega_p = \frac{\pi}{M}(1 - \rho)] \text{ and the attenuated band by } [\omega_s = \frac{\pi}{M}(1 + \rho),\ \pi]$$

- The passband ripple specifications referenced δ_1 and an attenuated band referenced δ_2 are generally the same for the sending and reception filters.

- At the pulsation π/M, generally the constraint $F_R(\pi/M) = F_T(\pi/M) = \sqrt{2}/2$ is dictated.

3 - The phase linearity

The choice of characteristic phase frequency filters that are perfectly linear is generally recommended for digital transmission systems [1, p. 325]. This especially has the advantage of preserving the instants of passage through zero of the transmitted binary trains.

4 - The matched pair of filters

In the case of linear modulation, if we assume that ISI is zero for the entire transmission system and that the channel noise is additive, white and Gaussian (BBAG), it is known that the matched pair, namely $F_T(z) = z^{-N} F_R(z^{-1})$ with $N = n_T = n_R$, the order of each filter, is optimal for the criterion of the signal-to-noise ratio (SNR) [2, pp. 51 sq.]. The SNR which is equal to $\frac{E}{N_0}\frac{1}{\|F_R\|^2\|F_T\|^2}$ [3] then reaches its maximum with $\|F_R\|^2\|F_T\|^2 = 1$.